

Solutions : les équations trigonométriques

Exercice 1

$$1) S = \left\{ \frac{p}{3} + k2p, -\frac{p}{3} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{3} \text{ et } \frac{5p}{3}$$

$$2) S = \left\{ -\frac{p}{4} + k2p, \frac{5p}{4} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{5p}{4} \text{ et } \frac{7p}{4}$$

$$3) S = \left\{ -\frac{p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{3p}{4} \text{ et } \frac{7p}{4}$$

$$4) S = \left\{ -\frac{p}{12} + kp, \frac{7p}{12} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{7p}{12}, \frac{11p}{12}, \frac{19p}{12} \text{ et } \frac{23p}{12}$$

$$5) S = \left\{ \frac{p}{12} + k\frac{p}{4}; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{12}, \frac{p}{3}, \frac{7p}{12}, \frac{5p}{6}, \frac{13p}{12}, \frac{4p}{3}, \frac{19p}{12} \text{ et } \frac{11p}{6}$$

$$6) S = \left\{ \frac{p}{4} + kp, \frac{3p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{4}, \frac{3p}{4}, \frac{5p}{4} \text{ et } \frac{7p}{4}$$

$$7) S = \emptyset$$

$$8) S = \{1,86 + k2p; -1,86 + k2p; k \in \mathbf{Z}\} \quad \text{et VP: } 1,86 \text{ et } 4,42$$

$$9) S = \left\{ \frac{2p}{5} + k2p, \frac{4p}{5} + k2p, \frac{6p}{5} + k2p, \frac{8p}{5} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{2p}{5}, \frac{4p}{5}, \frac{6p}{5} \text{ et } \frac{8p}{5}$$

$$10) S = \{-1,27 + kp; k \in \mathbf{Z}\} \quad \text{et VP: } 1,87 \text{ et } 5,01$$

$$11) S = \left\{ \frac{p}{6} + k\frac{2p}{5}, -\frac{p}{6} + k\frac{2p}{5}; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{6}, \frac{7p}{30}, \frac{17p}{30}, \frac{19p}{30}, \frac{29p}{30}, \frac{31p}{30}, \frac{41p}{30}, \frac{43p}{30}, \frac{53p}{30} \text{ et } \frac{11p}{6}$$

$$12) S = \left\{ \frac{p}{18} + k\frac{2p}{3}, -\frac{7p}{18} + k\frac{2p}{3}; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{18}, \frac{5p}{18}, \frac{13p}{18}, \frac{17p}{18}, \frac{25p}{18} \text{ et } \frac{29p}{18}$$

$$13) S = \left\{ -\frac{p}{12} + kp, -\frac{p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{3p}{4}, \frac{11p}{12}, \frac{7p}{4} \text{ et } \frac{23p}{12}$$

$$14) S = \left\{ k\frac{p}{2}; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{2}, p \text{ et } \frac{3p}{2}$$

$$15) S = \left\{ \frac{p}{3} + kp, -\frac{p}{3} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{3}, \frac{2p}{3}, \frac{4p}{3} \text{ et } \frac{5p}{3}$$

$$16) S = \{0,17 + kp; 1,4 + kp; k \in \mathbf{Z}\} \quad \text{et VP: } 0,17; 1,4; 3,31 \text{ et } 4,54$$

$$17) S = \left\{ k\frac{2p}{5}, -\frac{p}{3} + k\frac{2p}{3}; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{3}, \frac{2p}{5}, \frac{4p}{5}, p, \frac{6p}{5}, \frac{8p}{5} \text{ et } \frac{5p}{3}$$

$$18) S = \left\{ \frac{p}{10} + k\frac{p}{5}; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{10}, \frac{3p}{10}, \frac{p}{2}, \frac{7p}{10}, \frac{9p}{10}, \frac{11p}{10}, \frac{13p}{10}, \frac{3p}{2}, \frac{17p}{10} \text{ et } \frac{19p}{10}$$

$$19) S = \left\{ \frac{4p}{9} + k\frac{2p}{3}, -\frac{4p}{3} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{4p}{9}, \frac{2p}{3}, \frac{10p}{9} \text{ et } \frac{16p}{9}$$

- 20) $S = \left\{ \frac{5p}{48} + k\frac{p}{2}, \frac{13p}{24} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{5p}{48}, \frac{13p}{24}, \frac{29p}{48}, \frac{53p}{48}, \frac{37p}{24}$ et $\frac{77p}{48}$
- 21) $S = \left\{ \frac{p}{6} + k\frac{2p}{3}, \frac{p}{2} + k2p; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}, \frac{p}{2}, \frac{5p}{6}$ et $\frac{3p}{2}$
- 22) $S = \left\{ \frac{5p}{36} + k\frac{2p}{3}, -\frac{p}{12} + k2p; k \in \mathbf{Z} \right\}$ et VP: $\frac{5p}{36}, \frac{29p}{36}, \frac{53p}{36}$ et $\frac{23p}{12}$
- 23) $S = \left\{ \frac{p}{6} + k\frac{p}{2}, kp; k \in \mathbf{Z} \right\}$ et VP: $0, \frac{p}{6}, \frac{2p}{3}, p, \frac{7p}{6}$ et $\frac{5p}{3}$
- 24) $S = \left\{ \frac{p}{6} + k\frac{p}{3}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}, \frac{p}{2}, \frac{5p}{6}, \frac{7p}{6}, \frac{3p}{2}$ et $\frac{11p}{6}$
- 25) $S = \left\{ \frac{p}{3} + k\frac{p}{2}, \frac{p}{6} + k\frac{p}{2}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}, \frac{p}{3}, \frac{2p}{3}, \frac{5p}{6}, \frac{7p}{6}, \frac{4p}{3}, \frac{5p}{3}$ et $\frac{11p}{6}$
- 26) $S = \left\{ \frac{p}{3} + kp, -\frac{p}{3} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{3}, \frac{2p}{3}, \frac{4p}{3}$ et $\frac{5p}{3}$
- 27) $S = \left\{ \frac{p}{4} + k\frac{p}{2}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{4}, \frac{3p}{4}, \frac{5p}{4}$ et $\frac{7p}{4}$
- 28) $S = \left\{ \frac{p}{12} + kp, \frac{5p}{24} + k\frac{p}{2}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{12}, \frac{5p}{24}, \frac{17p}{24}, \frac{13p}{12}, \frac{29p}{24}$ et $\frac{41p}{24}$
- 29) $S = \left\{ \frac{p}{4} + kp, -\frac{p}{8} + k\frac{p}{2}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{4}, \frac{3p}{8}, \frac{7p}{8}, \frac{5p}{4}, \frac{11p}{8}$ et $\frac{15p}{8}$
- 30) $S = \left\{ \frac{p}{4} + kp, -\frac{p}{4} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{4}, \frac{3p}{4}, \frac{5p}{4}$ et $\frac{7p}{4}$

Exercice 2

- 1) $S = \left\{ k2p, \frac{2p}{3} + k2p, -\frac{2p}{3} + k2p; k \in \mathbf{Z} \right\} = \left\{ k\frac{2p}{3}; k \in \mathbf{Z} \right\}$ et VP: $0, \frac{2p}{3}$ et $\frac{4p}{3}$
- 2) $S = \left\{ \frac{2p}{3} + k2p, -\frac{2p}{3} + k2p; k \in \mathbf{Z} \right\}$ et VP: $\frac{2p}{3}$ et $\frac{4p}{3}$
- 3) $S = \left\{ \frac{p}{6} + k2p, \frac{5p}{6} + k2p; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}$ et $\frac{5p}{6}$
- 4) $S = \left\{ \frac{p}{6} + k2p, \frac{5p}{6} + k2p, -\frac{p}{6} + k2p, \frac{7p}{6} + k2p; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}, \frac{5p}{6}, \frac{7p}{6}$ et $\frac{11p}{6}$
- 5) $S = \{1, 32 + k2p; -1, 32 + k2p; p + k2p; k \in \mathbf{Z}\}$ et VP: $1, 32$; et $4, 97$
- 6) $S = \{2, 47 + k2p; -2, 47 + k2p; k \in \mathbf{Z}\}$ et VP: $2, 47$ et $3, 82$
- 7) $S = \{1, 25 + kp; 1, 11 + kp; k \in \mathbf{Z}\}$ et VP: $1, 11; 1, 25; 4, 25$ et 439
- 8) $S = \left\{ \frac{p}{3} + kp, \frac{p}{6} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}, \frac{p}{3}, \frac{7p}{6}$ et $\frac{4p}{3}$

$$9) S = \left\{ 1, 25 + kp, \frac{p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{4}; 1, 25; \frac{5p}{4} \text{ et } 4, 39$$

$$10) S = \left\{ \frac{p}{6} + kp, -\frac{p}{3} + kp; k \in \mathbf{Z} \right\} = \left\{ \frac{p}{6} + k\frac{p}{2}; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{6}, \frac{2p}{3}, \frac{7p}{6} \text{ et } \frac{5p}{3}$$

Exercice 3

$$1) S = \left\{ \frac{p}{2} + kp, \frac{p}{12} + kp, \frac{5p}{12} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{12}, \frac{5p}{12}, \frac{p}{2}, \frac{13p}{12}, \frac{17p}{12} \text{ et } \frac{3p}{2}$$

$$2) S = \left\{ k\frac{p}{3}, \frac{p}{3} + k2p, -\frac{p}{3} + k2p; k \in \mathbf{Z} \right\} = \left\{ k\frac{p}{3}; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{3}, \frac{2p}{3}, p, \frac{4p}{3} \text{ et } \frac{5p}{3}$$

$$3) S = \left\{ \frac{p}{4} + k\frac{p}{2}, \frac{p}{3} + k2p, -\frac{p}{3} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{4}, \frac{p}{3}, \frac{3p}{4}, \frac{5p}{4}, \frac{5p}{3} \text{ et } \frac{7p}{4}$$

$$4) S = \left\{ \frac{p}{5} + k\frac{2p}{5}, p + k4p; k \in \mathbf{Z} \right\} = \left\{ \frac{p}{5} + k\frac{2p}{5}; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{5}, \frac{3p}{5}, p, \frac{7p}{5} \text{ et } \frac{9p}{5}$$

$$5) S = \left\{ \frac{p}{2} + kp, \frac{p}{6} + k2p, \frac{5p}{6} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{6}, \frac{p}{2}, \frac{5p}{6} \text{ et } \frac{3p}{2}$$

$$6) S = \left\{ k\frac{p}{4}, \frac{p}{6} + k\frac{2p}{3}, -\frac{p}{2} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{6}, \frac{p}{4}, \frac{p}{2}, \frac{3p}{4}, \frac{5p}{6}, p, \frac{5p}{4}, \frac{3p}{2} \text{ et } \frac{7p}{4}$$

$$7) S = \left\{ k\frac{p}{2}, \frac{p}{3} + k2p, -\frac{p}{3} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{3}, \frac{p}{2}, p, \frac{3p}{2} \text{ et } \frac{5p}{3}$$

$$8) S = \left\{ \frac{p}{2} + kp, \frac{p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{4}, \frac{p}{2}, \frac{5p}{4} \text{ et } \frac{3p}{2}$$

$$9) S = \left\{ \frac{p}{4} + k\frac{p}{2}, kp, k\frac{p}{4}; k \in \mathbf{Z} \right\} = \left\{ k\frac{p}{4}; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{4}, \frac{p}{2}, \frac{3p}{4}, p, \frac{5p}{4}, \frac{3p}{2} \text{ et } \frac{7p}{4}$$

$$10) S = \left\{ k\frac{p}{3}; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0, \frac{p}{3}, \frac{2p}{3}, p, \frac{4p}{3} \text{ et } \frac{5p}{3}$$

Exercice 4

$$1) S = \left\{ 1, 107 + kp; \frac{p}{4} + kp; kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0; \frac{p}{4}; 1, 107; p, \frac{5p}{4} \text{ et } 4, 249$$

$$2) S = \left\{ -\frac{p}{6} + kp, \frac{p}{3} + kp, \frac{p}{2} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{3}, \frac{p}{2}, \frac{5p}{6}, \frac{4p}{3}, \frac{3p}{2} \text{ et } \frac{11p}{6}$$

$$3) S = \left\{ -\frac{p}{4} + kp, \frac{p}{4} + kp; k \in \mathbf{Z} \right\} = \left\{ \frac{p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{4}, \frac{3p}{4}, \frac{5p}{4} \text{ et } \frac{7p}{4}$$

$$4) S = \left\{ \frac{p}{3} + kp, -\frac{p}{3} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{3}, \frac{2p}{3}, \frac{4p}{3} \text{ et } \frac{5p}{3}$$

$$5) S = \{ 1, 107 + kp; 0, 644 + kp; k \in \mathbf{Z} \} \quad \text{et VP: } 0, 644; 1, 107; 3, 785 \text{ et } 4, 249$$

$$6) S = \left\{ \frac{p}{2} + kp; 0,588 + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0,588; \frac{p}{2}, 3,730 \text{ et } \frac{3p}{2}$$

$$7) S = \{0,927 + kp; 0,464 + kp; k \in \mathbf{Z}\} \quad \text{et VP: } 0,464; 0,927; 3,605 \text{ et } 4,069$$

$$8) S = \{1,093 + kp; -1,093 + kp; 0,478 + kp; -0,478 + kp; k \in \mathbf{Z}\}$$

$$\text{VP: } 0,478; 1,093; 2,048; 2,664; 3,619; 4,235; 5,190 \text{ et } 5,806$$

$$9) S = \{kp; k \in \mathbf{Z}\} \quad \text{et VP: } 0 \text{ et } p$$

$$10) S = \left\{ 0,955 + kp; -0,955 + kp; \frac{p}{4} + kp; -\frac{p}{4} + kp; k \in \mathbf{Z} \right\}$$

$$\text{VP: } \frac{p}{4}; 0,955; 2,186; \frac{3p}{4}; \frac{5p}{4}; 4,097; 5,328 \text{ et } \frac{7p}{4}$$

Exercice 5

$$1) S = \left\{ \frac{p}{12} + k2p, \frac{7p}{12} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{12} \text{ et } \frac{7p}{12}$$

$$2) S = \left\{ \frac{p}{2} + k2p, \frac{p}{6} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{6} \text{ et } \frac{p}{2}$$

$$3) S = \left\{ -\frac{p}{12} + k2p, \frac{7p}{12} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{7p}{12} \text{ et } \frac{23p}{12}$$

$$4) S = \left\{ \frac{p}{8} + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{8} \text{ et } \frac{9p}{8}$$

$$5) S = \left\{ \frac{p}{2} + k2p; -0,395 + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{p}{2} \text{ et } 5,889$$

$$6) S = \{1,878 + k2p; -0,702 + k2p; k \in \mathbf{Z}\} \quad \text{et VP: } 1,878 \text{ et } 5,581$$

$$7) S = \{-p + k2p; 2,651 + k2p; k \in \mathbf{Z}\} \quad \text{et VP: } 2,651 \text{ et } p$$

$$8) S = \left\{ -\frac{7p}{12} + k2p, \frac{11p}{12} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{11p}{12} \text{ et } \frac{17p}{12}$$

$$9) S = \left\{ -\frac{p}{4} + kp; -1,768 + kp; k \in \mathbf{Z} \right\} \quad \text{et VP: } 1,373; \frac{3p}{4}; 4,515 \text{ et } \frac{7p}{4}$$

$$10) S = \left\{ -\frac{p}{2} + k2p; p + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } p \text{ et } \frac{3p}{2}$$

Exercices récapitulatifs

$$1) S = \left\{ \frac{2p}{3} + k2p; k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } 0 \text{ et } \frac{2p}{3}$$

$$2) S = \left\{ -\frac{p}{6} + k2p, \frac{7p}{6} + k2p; k \in \mathbf{Z} \right\} \quad \text{et VP: } \frac{7p}{6} \text{ et } \frac{11p}{6}$$

- 3) $S = \left\{ \frac{p}{4} + kp; -0,197 + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{4}; 2,944; \frac{5p}{4}$ et 6,086
- 4) $S = \left\{ \frac{p}{6} + kp, \frac{5p}{6} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{6}, \frac{5p}{6}, \frac{7p}{6}$ et $\frac{11p}{6}$
- 5) $S = \left\{ kp, \frac{p}{3} + kp; k \in \mathbf{Z} \right\}$ et VP: $0, \frac{p}{3}, p$ et $\frac{4p}{3}$
- 6) $S = \left\{ \frac{p}{8} + k\frac{p}{4}, \frac{p}{6} + k\frac{p}{3}; k \in \mathbf{Z} \right\}$
 VP: $\frac{p}{8}, \frac{p}{6}, \frac{3p}{8}, \frac{p}{2}, \frac{5p}{8}, \frac{5p}{6}, \frac{7p}{8}, \frac{9p}{8}, \frac{7p}{6}, \frac{11p}{8}, \frac{3p}{2}, \frac{13p}{8}, \frac{11p}{6}$ et $\frac{15p}{8}$
- 7) $S = \left\{ \frac{p}{2} + kp; 0,685 + kp; 2,457 + kp; k \in \mathbf{Z} \right\}$ et VP: $0,685; \frac{p}{2}; 2,457; 3,826; \frac{3p}{2}$ et 5,598
- 8) $S = \{-0,292 + k2p; -2,610 + k2p; k \in \mathbf{Z}\}$ et VP: 3,673 et 5,992
- 9) $S = \left\{ \frac{p}{2} + k2p, -\frac{p}{6} + k\frac{2p}{3}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{2}, \frac{7p}{6}$ et $\frac{11p}{6}$
- 10) $S = \left\{ k\frac{p}{4}; k \in \mathbf{Z} \right\}$ et VP: $0, \frac{p}{4}, \frac{p}{2}, \frac{3p}{4}, p, \frac{5p}{4}, \frac{3p}{2}$ et $\frac{7p}{4}$
- 11) $S = \left\{ \frac{p}{2} + k2p, \frac{3p}{10} + k\frac{2p}{5}; k \in \mathbf{Z} \right\}$ et VP: $\frac{3p}{10}, \frac{p}{2}, \frac{7p}{10}, \frac{11p}{10}, \frac{3p}{2}$ et $\frac{19p}{10}$
- 12) $S = \left\{ \frac{p}{36} + k\frac{p}{3}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{36}, \frac{13p}{36}, \frac{25p}{36}, \frac{37p}{36}, \frac{49p}{36}$ et $\frac{61p}{36}$
- 13) $S = \left\{ kp, \frac{p}{2} + kp, k\frac{2p}{3}; k \in \mathbf{Z} \right\}$ et VP: $0, \frac{p}{2}, \frac{2p}{3}, p, \frac{4p}{3}$ et $\frac{3p}{2}$
- 14) $S = \left\{ kp, \frac{p}{4} + k\frac{p}{2}, k\frac{p}{3}; k \in \mathbf{Z} \right\}$ et VP: $0, \frac{p}{4}, \frac{p}{3}, \frac{2p}{3}, \frac{3p}{4}, p, \frac{5p}{4}, \frac{4p}{3}, \frac{5p}{3}$ et $\frac{7p}{4}$
- 15) $S = \{-1,093 + kp; 1,093 + kp; -0,478 + kp; 0,478 + kp; k \in \mathbf{Z}\}$
 VP: 0,478; 1,093; 2,048; 2,664; 3,620; 4,235; 5,190 et 5,806
- 16) $S = \left\{ \frac{p}{2} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{2}$ et $\frac{3p}{2}$
- 17) $S = \left\{ \frac{p}{4} + k\frac{p}{2}, \frac{p}{10} + k\frac{2p}{5}, \frac{p}{6} + k\frac{2p}{3}; k \in \mathbf{Z} \right\}$
 VP: $\frac{p}{10}, \frac{p}{6}, \frac{p}{4}, \frac{p}{2}, \frac{3p}{4}, \frac{5p}{6}, \frac{9p}{10}, \frac{5p}{4}, \frac{13p}{10}, \frac{3p}{2}, \frac{17p}{10}$ et $\frac{7p}{4}$
- 18) $S = \left\{ \frac{p}{4} + k2p; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{4}$
- 19) $S = \{0,948 + k2p; -2,803 + k2p; k \in \mathbf{Z}\}$ et VP: 0,948 et 3,480
- 20) $S = \left\{ -\frac{p}{12} + kp, -\frac{5p}{12} + kp; k \in \mathbf{Z} \right\}$ et VP: $\frac{7p}{12}, \frac{11p}{12}, \frac{19p}{12}$ et $\frac{23p}{12}$
- 21) $S = \left\{ \frac{p}{10} + k\frac{2p}{5}, -\frac{p}{6} + k\frac{2p}{3}; k \in \mathbf{Z} \right\}$ et VP: $\frac{p}{10}, \frac{p}{2}, \frac{9p}{10}, \frac{7p}{6}, \frac{13p}{10}, \frac{17p}{10}$ et $\frac{11p}{6}$

$$22) S = \{kp; k \in \mathbf{Z}\} \quad \text{et} \quad VP: 0 \text{ et } p$$

$$23) S = \left\{ \frac{p}{4} + k \frac{2p}{3}, -\frac{3p}{4} + k 2p; k \in \mathbf{Z} \right\} \quad \text{et} \quad VP: \frac{p}{4}, \frac{11p}{12}, \frac{5p}{4} \text{ et } \frac{19p}{12}$$

$$24) S = \left\{ \frac{p}{6} + k \frac{2p}{3}, k \frac{p}{2}; k \in \mathbf{Z} \right\} \quad \text{et} \quad VP: 0, \frac{p}{6}, \frac{p}{2}, \frac{5p}{6}, p \text{ et } \frac{3p}{2}$$

$$25) S = \left\{ -\frac{p}{12} + kp, -\frac{p}{4} + kp; k \in \mathbf{Z} \right\} \quad \text{et} \quad VP: \frac{3p}{4}, \frac{11p}{12}, \frac{7p}{4} \text{ et } \frac{23p}{12}$$

$$26) S = \left\{ kp, \frac{p}{8} + k \frac{p}{4}, k \frac{p}{3}; k \in \mathbf{Z} \right\}$$

$$VP: 0, \frac{p}{8}, \frac{p}{3}, \frac{3p}{8}, \frac{5p}{8}, \frac{2p}{3}, \frac{7p}{8}, p, \frac{9p}{8}, \frac{4p}{3}, \frac{11p}{8}, \frac{13p}{8}, \frac{5p}{3} \text{ et } \frac{15p}{8}$$

$$27) S = \{2,554 + kp; k \in \mathbf{Z}\} \quad \text{et} \quad VP: 2,554 \text{ et } 5,695$$

$$28) S = \left\{ k \frac{2p}{3}; k \in \mathbf{Z} \right\} \quad \text{et} \quad VP: 0, \frac{2p}{3} \text{ et } \frac{4p}{3}$$

$$29) S = \left\{ \frac{2p}{15} + k \frac{p}{2}; k \in \mathbf{Z} \right\} \quad \text{et} \quad VP: \frac{2p}{15}, \frac{19p}{30}, \frac{17p}{15} \text{ et } \frac{49p}{30}$$

$$30) S = \{1,082 + k 2p; 2,060 + k 2p; k \in \mathbf{Z}\} \quad \text{et} \quad VP: 1,082 \text{ et } 2,060$$