TRIGONOMETRIE : équations trigonométriques

Contrôle-série N°1 (2012-2013)

Enoncés

$$\tan^2 x + \frac{1}{2\cos^2 x} = \frac{-5}{2\cos x}$$

$$\cos 3x - \cos 5x = 2\sin 4x$$

$$\sin 3x = \cos(\frac{\pi}{5} - x)$$

$$\cos x - \sin x = \frac{\sqrt{2}}{2}$$

Solutions

$$\tan^2 x + \frac{1}{2\cos^2 x} = \frac{-5}{2\cos x}$$

$$CE : x \neq \frac{\pi}{2} + k\pi$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{1}{2\cos^2 x} = \frac{-5}{2\cos x}$$

$$2\sin^2 x + 5\cos x + 1 = 0$$

par FF, on a

$$2(1 - \cos^2 x) + 5\cos x + 1 = 0$$

$$-2\cos^2 x + 5\cos x + 3 = 0$$

par Δ*, on a*

$$\sin x = 3 (\grave{a} \ \ rejeter) \ \ ou \quad \sin x = -\frac{1}{2}$$

$$\sin x = \sin(-\frac{\pi}{6})$$

$$x = -\frac{\pi}{6} + k2\pi \quad ou \quad x = \pi - (-\frac{\pi}{6}) + k2\pi = \frac{7\pi}{6} + k2\pi$$

$$S_p = \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$\cos 3x - \cos 5x = 2\sin 4x$

par les formules de Simpson :

$-2\sin 4x \sin(-x) = 2\sin 4x$

$2\sin 4x \sin x - 2\sin 4x = 0$

$2\sin 4x(\sin x - 1) = 0$

$\sin 4x = 0 \quad ou \quad \sin x = 1$

$x = \frac{k\pi}{4} \qquad\qquad x = \frac{\pi}{2} + k2\pi$

$S_p = \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$

$\sin 3x = \cos(\frac{\pi}{5} - x)$

par les angles complémentaires :

$\sin 3x = \sin(\frac{\pi}{2} - (\frac{\pi}{5} - x))$

$\sin 3x = \sin(\frac{3\pi}{10} + x)$

$3x = \frac{3\pi}{10} + x + k2\pi \quad ou \quad 3x = \pi - (\frac{3\pi}{10} + x) + k2\pi$

$x = \frac{3\pi}{20} + k\pi \qquad\qquad x = \frac{7\pi}{40} + k\frac{\pi}{2}$

$S_p = \left\{\frac{3\pi}{20}, \frac{23\pi}{20}, \frac{7\pi}{40}, \frac{27\pi}{40}, \frac{47\pi}{40}, \frac{67\pi}{40}\right\}$

$\cos x - \sin x = \frac{\sqrt{2}}{2}$

équation de type $a\cos x + b\sin x = c$

$\exists \varphi : \tan\varphi = 1$

$\cos x - \tan\frac{\pi}{4}\sin x = \frac{\sqrt{2}}{2}$

$\cos x - \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}\sin x = \frac{\sqrt{2}}{2}$

$\cos x \cos\frac{\pi}{4} - \sin\frac{\pi}{4}\sin x = \frac{\sqrt{2}}{2}\cos\frac{\pi}{4}$

$\cos(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = \frac{1}{2}$

$\cos(x + \frac{\pi}{4}) = \cos\frac{\pi}{3}$

$x + \frac{\pi}{4} = \frac{\pi}{3} + k2\pi \quad ou \quad x + \frac{\pi}{4} = -\frac{\pi}{3} + k2\pi$

$x = \frac{\pi}{12} + k2\pi \qquad\qquad x = -\frac{7\pi}{12} + k2\pi$

$S_p = \left\{\frac{\pi}{12}, \frac{17\pi}{12}\right\}$

Contrôle-série N°2 (2012-2013)

Enoncés

$$2\cot^2 x - \frac{5}{\sin^2 x} = \frac{7}{\sin x}$$

$$\sin 3x - \sin 5x = 2\sin x$$

$$\cos 3x = \sin(\frac{\pi}{5} - x)$$

$$\cos x + \sin x = \frac{-\sqrt{2}}{2}$$

Solutions

$$2\cot^2 x - \frac{5}{\sin^2 x} = \frac{7}{\sin x}$$

$$CE : x \neq \pi + k\pi$$

$$\frac{2\cos^2 x}{\sin^2 x} - \frac{5}{\sin^2 x} = \frac{7}{\sin x}$$

$$2\cos^2 x - 7\sin x - 5 = 0$$

$$par\ \ FF, on\ a$$

$$2(1 - \sin^2 x) - 7\sin x - 5 = 0$$

$$-2\sin^2 x - 7\sin x - 3 = 0$$

$$par\ \Delta, on\ a$$

$$\sin x = -3(\grave{a}\ \ rejeter)\ \ ou\ \ \ \sin x = -\frac{1}{2}$$

$$\sin x = \sin(-\frac{\pi}{6})$$

$$x = -\frac{\pi}{6} + k2\pi\ \ \ ou\ \ \ x = \pi - (-\frac{\pi}{6}) + k2\pi = \frac{7\pi}{6} + k2\pi$$

$$S_p = \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$$

$$\sin 3x - \sin 5x = 2\sin x$$

$$par\, les\ \ formules\ \ de\ \ Simpson :$$

$$2\sin(-x)\cos 4x = 2\sin x$$

$$-2\sin x\cos 4x - 2\sin x = 0$$

$$-2\sin x(\cos 4x + 1) = 0$$

$$\sin x = 0\ \ \ ou\ \ \ \cos 4x = -1$$

$$x = k\pi \qquad\qquad x = \frac{\pi}{4} + k\frac{\pi}{2}$$

$$S_p = \left\{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

$$\cos 3x = \sin(\frac{\pi}{5} - x)$$

par les angles complémentaires :

$$\cos 3x = \cos(\frac{\pi}{2} - (\frac{\pi}{5} - x))$$

$$\cos 3x = \cos(\frac{3\pi}{10} + x)$$

$$3x = \frac{3\pi}{10} + x + k2\pi \quad ou \quad 3x = -(\frac{3\pi}{10} + x) + k2\pi$$

$$x = \frac{3\pi}{20} + k\pi \qquad\qquad x = -\frac{3\pi}{40} + k\frac{\pi}{2}$$

$$S_p = \left\{ \frac{3\pi}{20}, \frac{23\pi}{20}, \frac{17\pi}{40}, \frac{37\pi}{40}, \frac{57\pi}{40}, \frac{77\pi}{40} \right\}$$

$$\cos x + \sin x = -\frac{\sqrt{2}}{2}$$

équation de type $a\cos x + b\sin x = c$

$$\exists \varphi : \tan\varphi = 1$$

$$\cos x + \tan\frac{\pi}{4}\sin x = -\frac{\sqrt{2}}{2}$$

$$\cos x + \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}\sin x = -\frac{\sqrt{2}}{2}$$

$$\cos x\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\sin x = -\frac{\sqrt{2}}{2}\cos\frac{\pi}{4}$$

$$\cos(x - \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = -\frac{1}{2}$$

$$\cos(x - \frac{\pi}{4}) = \cos\frac{2\pi}{3}$$

$$x - \frac{\pi}{4} = \frac{2\pi}{3} + k2\pi \quad ou \quad x - \frac{\pi}{4} = -\frac{2\pi}{3} + k2\pi$$

$$x = \frac{11\pi}{12} + k2\pi \qquad\qquad x = -\frac{5\pi}{12} + k2\pi$$

$$S_p = \left\{ \frac{11\pi}{12}, \frac{19\pi}{12} \right\}$$