

ANALYSE : Fonctions cyclométriques

Etudes de fonctions (énoncés du syllabus)

$$f(x) = \arctg \frac{1}{2-x}$$

$$D = \mathbb{R} \setminus \{2\} \quad I =]-\frac{\pi}{2}, \frac{\pi}{2}[$$

Inters. avec OY (0 ; arctg 0,5)

Inters. avec OX /

$$\begin{array}{c} x \\ f(x) \end{array} \quad \begin{array}{c} 2 \\ + \quad / \quad - \end{array}$$

ni pair, ni impair

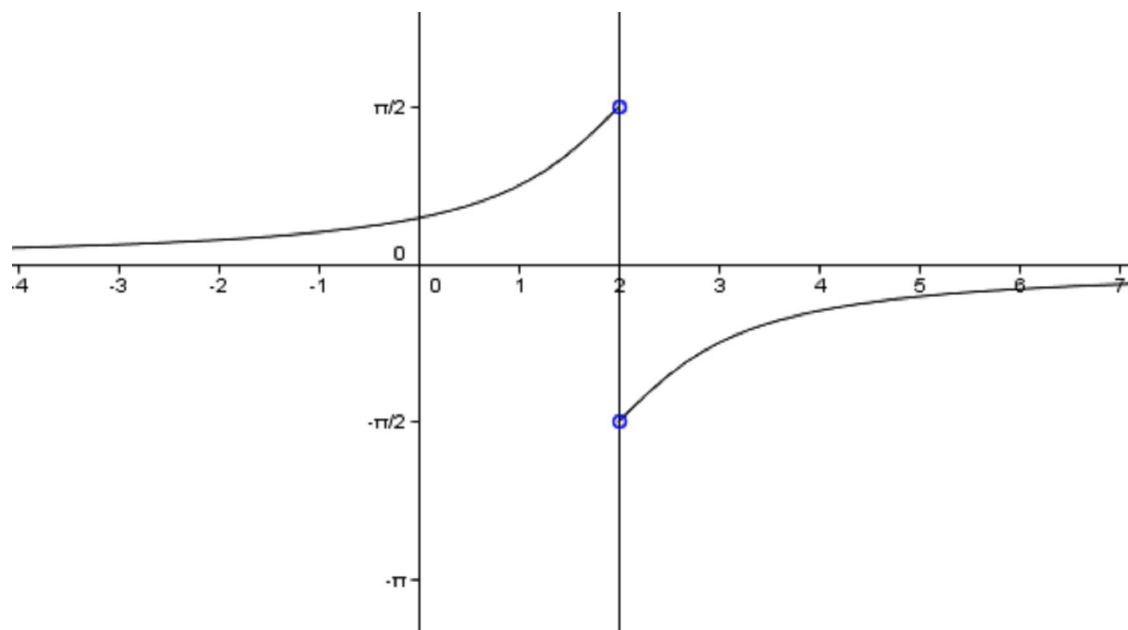
$$\lim_{x \rightarrow 2^\pm} f(x) = \mp \frac{\pi}{2}$$

$$\lim_{x \rightarrow \pm\infty} f(x) = 0^\mp \quad A.H. \equiv y = 0$$

$$f'(x) = \frac{1}{1 + \left(\frac{1}{2-x}\right)^2} \frac{-(-1)}{(2-x)^2} = \frac{1}{x^2 - 4x + 5} > 0$$

$$f''(x) = \frac{-(2x-4)}{(x^2-4x+5)^2}$$

$$\begin{array}{c} x \\ f'(x) \end{array} \quad \begin{array}{c} 2 \\ + \quad / \quad - \end{array}$$



$$f(x) = \arctan \frac{2x}{1-x^2}$$

$$D = \mathbb{R} \setminus \{-1, 1\} \quad I =]-\frac{\pi}{2}, \frac{\pi}{2}[$$

Inters. avec OY (0 ; 0)

Inters. avec OX (0 ; 0)

impair

x	-1	0	1
$f(x)$	$+$	$-$	$+$

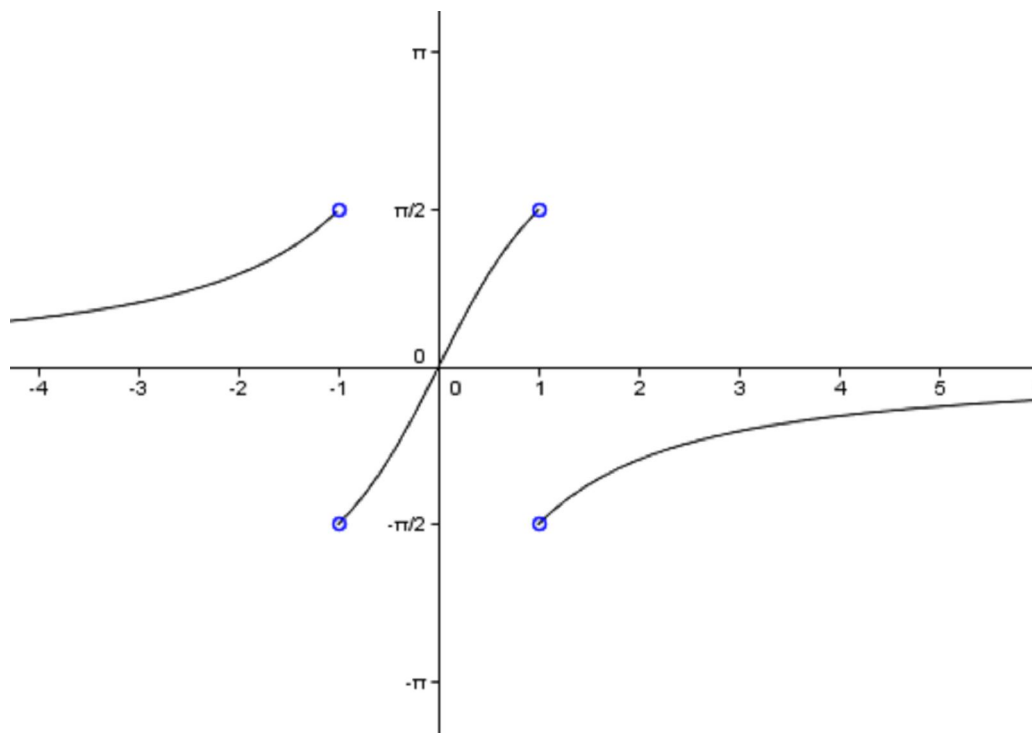
$$\lim_{x \rightarrow \pm 1} f(x) = \mp \frac{\pi}{2}$$

$$\lim_{x \rightarrow +\infty} f(x) = 0^- \quad A.H. \equiv y = 0$$

$$f'(x) = \frac{1}{1 + \left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{2(1-x^2) - 2x(-2x)}{(1-x^2)^2} = \frac{2(x^2+1)}{x^4 + 2x^2 + 1} = \frac{2}{x^2+1} > 0$$

$$f''(x) = \frac{-4x}{(x^2+1)^2}$$

x	0
$f''(x)$	$-$



$$f(x) = x - \arctan(x)$$

$$D = \mathbb{R} \quad I = \mathbb{R}$$

Inters. avec OY (0 ; 0)

Inters. avec OX (0 ; 0)

impair

$$\begin{array}{ccc} x & & 0 \\ f(x) & - & 0 \end{array} \quad +$$

$$\lim_{x \rightarrow +\infty} f(x) = "+\infty - \frac{\pi}{2}" = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} 1 - \frac{\arctan(x)}{x} = 1 - \lim_{x \rightarrow +\infty} \frac{1}{1+x^2} = 1$$

$$\lim_{x \rightarrow +\infty} (x - \arctan(x) - x) = \lim_{x \rightarrow +\infty} (-\arctan(x)) = -\frac{\pi}{2}$$

$$A.O. \equiv y = x - \frac{\pi}{2} \text{ en } +\infty$$

$$A.O. \equiv y = x + \frac{\pi}{2} \text{ en } -\infty$$

$$f'(x) = 1 - \frac{1}{1+x^2} = \frac{x^2}{1+x^2} > 0$$

$$f''(x) = \frac{2x(1+x^2) - x^2 \cdot 2x}{(1+x^2)^2} = \frac{2x}{(1+x^2)^2}$$

$$\begin{array}{ccc} x & & 0 \\ f'(x) & - & \text{PI} \end{array} \quad +$$

$$t_{\text{PI}} \equiv y = 0$$

